

Why IRR is Not the Rate of Return for Your Investment: Introducing AIRR to the Real Estate Community

Executive Summary. *The internal rate of return (IRR) is used extensively in the real estate sector. Unfortunately, the IRR calculation itself assumes interim investment values that are mechanically generated by the IRR equation itself and will almost surely differ from the true interim values of the project under consideration. To the extent that these values differ, the IRR result will not be an accurate rate of return. Furthermore, from an ex post, i.e., performance reporting standpoint, such values implied by the IRR will almost certainly contradict any estimated project values being used for time-weighted rate of return (TWR) purposes. A new metric called Average IRR (AIRR) produces a correct money-weighted rate of return (MWR) for a project. Furthermore, AIRR has none of the other problems that the IRR has (e.g., it always exists and is unique), and it appropriately accounts for the amounts actually invested over the course of the investment.*

by Dean Altshuler*
Carlo Alberto Magni**

Discounted cash flow (DCF) methodology plays a major role in the real estate arena, and is used by both academics and practitioners. Its use is commonplace, both in regard to ex ante investment decision-making, as well as ex post investment evaluation (the latter is commonly referred to as “performance measurement”). The two main elements of DCF methodology are the sister functions of net present value (NPV) and internal rate of return (IRR), where the IRR is the rate of return that sets the NPV to zero.

The NPV is favored among academics. Yung and Sherman (1995, p. 18) state “For abstruse reasons that are beyond the scope of this article, college professors prefer NPV analysis over IRR analysis. But all surveys indicate that lenders find it more appealing to analyze potential investments in terms of percentage rates of return than by comparing dollars of NPV.”

Moreover, non-lender practitioners also, by far, prefer to focus on the IRR (Gitman and Forrester, 1977; Stanley and Block, 1984; Boyd, MacGillivray, and Schwartz, 1995; Farragher and Kleiman, 1996; Burns and Walker, 1997). No doubt, practitioners find the notion of a rate of return more intuitively compelling than the notion of the value being added for the investor (Evans and Forbes, 1993). Therefore, even when economic profitability might be better obtained in the form of a present value analysis, it is the IRR that is far more often favored. Even more so in the real estate arena, the

*Bard Consulting LLC, Manhattan Beach, CA 90266 or deana@bardconsulting.com.

**University of Modena, Reggio Emilia, Italy or magni@unimo.it.

IRR holds a sacred position with regard to decisions about acquisition, as well as choices between mutually exclusive investments. The IRR also predominates in terms of determining performance-based fees for real estate funds and ventures (Carey, 2003; Altshuler and Schneiderman, 2011), and even has a larger role in performance measurement and attribution (Geltner, 2003) than is commonly seen with other asset classes, where time-weighted rate of return methods tend to predominate more.

Since the era of its creation by Fisher (1930), Boulding (1935), and Keynes (1936), a considerable body of literature has been devoted to the IRR and to its biases in economic theory, business finance, management science, and engineering economy [see Magni (2010) for a recent review]. Even within the literature of real estate, an asset class that, in this regard, is less rigorous than other asset classes (Jaffe, 1977), the IRR has not managed to escape the scrutiny of scholars unscathed. Brown (2006) addressed several problems of the IRR: the multiple IRR solution problem, the ranking problem, the investment scale problem, and the simulation problem (Jensen's inequality). The meaning of the IRR and its alleged reinvestment assumption was explored by Spies (1983), who interpreted the IRR as a period rate applied to the internal capital amounts invested in the project, thereby clarifying that the IRR is earned on the capital employed in the asset, a point that was also suggested some years earlier by Akerson (1976) and, more recently, again by Crean (2005). Colwell (1995) addressed the multiple IRR problem and distinguished between relevant and irrelevant IRRs. Chang and Owens (1999), Sherman and Walters (1997), and Messner and Findlay (1975), the last article notwithstanding a critique by Young (1979), suggested a modification of the IRR to circumvent its flaws.¹

Less acknowledged, in the world of real estate investment, is the manner in which IRR results are cited without recognition of an appropriate benchmark (i.e., a cost of capital). "Our results show that the IRR is a powerful tool for measuring the expected return and examining its variations"

(Fisher and Goetzmann, 2005, p. 44), yet it is noteworthy that the authors do highlight the need for a benchmark return to confront the IRR with. Marrs and Tomlinson (2005) also supported the use of the IRR as opposed to the time-weighted rate of return (TWR), but then warned that the use of the IRR should not abstract from the explicit determination of the investor's cost of capital. "IRRs should never be used, whether in evaluating the performance of an investment or an investment's manager, without an appropriate understanding of an investor's cost of capital" (p. 6).

The cost of capital is the minimum required return and its determination is important to assess the economic profitability of an investment, whatever the purpose of the analysis (Riggs and Harms, 2000; Geltner, 2003; Breidenbach, Mueller, and Schulte, 2006; Liapis, Christofakis, and Papacharalampous, 2011). Indeed, the proliferation of IRR statistics, unaccompanied by appropriately risk-adjusted benchmarks, is a sad legacy of real estate performance reporting to date. A money-weighted rate of return metric that *explicitly* requires a determination of the cost of capital would seem to be a good idea.

Nevertheless, the IRR reigns supreme as the pre-eminent tool utilized in the arena of real estate investment decisions. And it would seem that things have only gotten better for IRR proponents recently. Specifically, the past decade has witnessed a pretty remarkable advance in research dispelling one of the alleged deficiencies of the IRR, an advance that has passed unnoticed even by most real estate scholars, not to mention practitioners. In particular, Hazen (2003) made use of the not-so-well-known fact that the IRR equation itself assumes a sequence of interim values for the project that the IRR is earned on. Hazen showed that this sequence of values, in turn, uniquely determines the overall capital invested in the project. In this way, he was able to show that, when a cash flow sequence produces multiple IRR solutions, each solution is reconcilable with the others, as it is earned on a different set of interim values for the project (and, therefore, on a different overall capital amount). Specifically, by multiplying the overall implied capital by the spread between the

IRR and the cost of capital, one obtains the exact same NPV result, no matter which of the multiple IRR solutions is used.

Although this finding is impressive and could win over some IRR skeptics, the requisite analysis underscores an “inconvenient truth” about the IRR, one that seems to have gone largely overlooked for decades by scholars and practitioners. And it is that the IRR ‘internally’ devises its own implied sequence of interim project values upon which it is a rate of return and, therefore, ‘automatically’ determines the overall implied capital that it ‘believes’ is invested in a project.

However, simple logic would seem to dictate that, if the IRR is to be a valid rate of return for a project, then the IRR’s ‘internally’ implied values *must* be accurate estimates of the true interim values of that project. In what follows, we argue that they will not be; they are arbitrary estimates of value. We introduce a new metric named Average IRR (AIRR) that solves this problem, not to mention virtually all of the other problems of the IRR. The AIRR, introduced in Magni (2010), can be used for both ex ante and ex post evaluation, just like the IRR, but, unlike the IRR, it does not require solving a complex polynomial equation, for it is based on the simple notion that the rate of return of an investment is just what it is supposed to be: the ratio of the return obtained by the asset to the overall capital invested into it. Equivalently, AIRR can be viewed as the mean of holding period rates of return weighted by the present value of the interim amounts invested.

The structure of this paper is as follows. The first section briefly describes Hazen’s (2003) answer to the multiple IRR problem, but then proceeds to detail how the solution to the multiple IRR problem inadvertently exposes the IRR’s link to an implicit overall capital that has nothing to do with the actual capital still invested in the asset. The second section introduces the AIRR, which overcomes this problem by allowing the use of interim values that are deliberately estimated. Finally, the third section includes some concluding remarks addressing the relative merits of the AIRR as compared to the IRR. Most of the equations and examples are in the appendices.

For Better and For Worse: Implications of the Last Decade’s Research on the IRR

The NPV is almost universally considered the “gold standard” in determining whether an investment is worth undertaking. Finance theorists, and even a handful of practitioners, have long espoused that the IRR suffers from several flaws, among which are its inconsistency with NPV for accept/reject decisions, as well as its project rankings. In particular, when the IRR produces multiple solutions, the solutions are irreconcilable. Although we believe that multiple IRR solutions occur much more frequently in theory than in real estate practice, nevertheless, this has been one of the *conceptual* roadblocks for IRR as a reliable money-weighted rate of return.

The Good News

As it turns out, the multiple IRR issue is not problematic any longer, as a result of research conducted in the last decade (Hazen, 2003). However, it is not clear that many real estate scholars and IRR users are aware of this, since it can take a long time for important scholarly research to find its way into the mainstream of investing practice.

Specifically, in the referenced article, Hazen investigated cash flow streams that have multiple IRR solutions.² For each solution, he derived what we will call the “implied sequence of interim project values,” which are the period-by-period undistributed remaining amounts still invested in the project; such sequence determines the overall capital invested in the project (the present value of the sequence of interim project values) and the IRR simply represents the rate of return *on* such overall capital.³

Although the existence of an implied sequence of interim project values may seem superfluous, it is fundamental to the very definition of the IRR. The IRR was first devised (and named “internal”) by Boulding (1935),⁴ who began his derivation with such a sequence of interim values, generated by assuming that returns are generated at a constant rate: the IRR itself. Details of his derivation are

provided in Appendix 1 (see equations (A.1)–(A.3) for the algebraic relationship between rate of return and implied interim project values).

Real estate appraiser Charlie Akerson introduced the implied interim value concept to real estate practitioners in a treatise on the IRR (Akerson, 1976). In particular, he demonstrated the technique for computing the sequence of interim project values implied by the IRR by making a savings account analogy for the IRR where the implied project values are simply the ending balances, each period, of the savings account (the initial project value is merely the amount of the initial contribution). That value is then grown at the IRR rate and, to that result, is subtracted the subsequent period's net distribution (so a contribution is, in effect, added), which produces the next period's beginning project value. The process repeats itself until the last period where, by definition of IRR, the ending project value must and will end up at zero. This is the same procedure that was introduced by Boulding (1935), although expressed in less formal terms. This procedure was essentially repeated in later years by both Spies (1983) and Crean (2005).

Using this same methodology, but applying it to cash flow sequences that have multiple IRR solutions, Hazen's (2003) analysis necessarily produced a different sequence of interim project values for each of the IRR solutions computed in his examples. In this manner, he underscored the fact that each IRR solution is a "rate of return" only in reference to a companion sequence of interim project values that each IRR solution itself produces. Hazen thereby rightfully argued that the differences among multiple IRR solutions, when they exist, are perfectly reconcilable (i.e., each IRR solution is a distinct rate of return applying to a distinct sequence of interim project values).

Further, Hazen (2003) cleverly noticed that any one IRR solution may be used for making accept/reject decisions, by checking whether the overall capital's NPV is positive or negative: if it is positive, then the IRR is a rate of earnings (i.e., the project is a 'net investment') and the project is profitable if and only if the IRR exceeds the cost of capital; however, if it is negative, then the IRR is a

rate of cost (i.e., the project is a 'net borrowing') and the project is profitable if and only if the IRR is smaller than the COC. In such a way, multiple IRRs and NPV are reconciled [see Hazen (2003) for details].

The Not So Good News

On face value, the news that the IRR is devoid of this critical flaw is exciting. Nevertheless, at least from our vantage point, any excitement is more than offset by the realization of an 'inconvenient truth,' which is that we are still left to ponder the most basic of questions: "Which of the IRR solutions is appropriate/correct for a project?" It seems that the answer is "almost surely, none of them" for the following reason: the IRR is a meaningful rate of return only if the sequence of true (i.e., marketplace) interim values of a project actually equals the specific sequence of interim values implied by the IRR calculation. The likelihood that such is the case is virtually nil—and if it did occur, it would be a result of mere happenstance. Simply put, the IRR 'internally' derives its own sequence of interim project values upon which it is a valid rate of return. However, there is no good reason to conclude that these internally-derived values reflect the reality of how the project's value actually grows, and/or declines, over time. This conclusion, although inadvertently prompted by an analysis of a cash flow stream with multiple IRR solutions, is appropriate for all cash flow streams, including those with only a single IRR solution. In other words: the IRR is surely the wrong rate of return even in the single IRR solution cases that have always been considered non-problematic.

Worse yet, given that the IRR result is determined by all of the investment's cash flows, it follows that any interim value implied by that IRR result is determined, in part, by *past* cash flows. However, this contradicts a fundamental investment precept: the value of an investment only depends on *future* cash flows. Therefore, by definition, the IRR-implied interim values cannot represent the true marketplace values, for the latter only depends on expectations, not on past cash flows (see Appendix 1). It is thus illogical to use the IRR, because its assumed interim values, the very ones upon which the computed IRR is earned, are contrary to fact.

AIRR: The Remedy for the IRR's Ills

We have argued that the interim values implied by the IRR are poor indicators of true value. So, the question arises as to whether there is any scenario where the IRR's values, and hence, the IRR result itself, is adequate. Only two scenarios come to mind as potential candidates. The first scenario is when we simply have no decent estimates as to what our project's actual interim valuations are, and so are willing to assume that they are equal to the IRR function's implied values, simply due to lack of a viable alternative. The second scenario is when we have some comfort that the IRR result, while incorrect, is still probably "close enough" to the correct rate of return for our project.

With regard to scenario 1, it seems reasonable to suggest that any rational estimate of interim values, even if made under the most trying of circumstances, must produce a result that is superior to what would be mathematically implied by the arbitrary, always constant, rate of return assumption of the IRR; the latter uses an algorithm that can know nothing about the nuances of the investment in question, not to mention the market that drives it. Nevertheless, it would be a disservice to ignore the sizable cadre of practitioners who are simply unwilling to undertake 'value estimation under considerable uncertainty'; analysts who, in fact, embrace the IRR, partly for the very reason that it allows them to forego this difficult task. Unless they are simply lazy, the members of this cadre probably do not realize that there is a very straightforward procedure (one that is the procedure of favor for many analysts), which completely automates the determination of interim values in a market-consistent manner, one that is superior to the IRR's flawed approach. The procedure is, quite simply:

- When looking forward, compute each projected interim value as the NPV of future estimated cash flows.
- When looking backward, compute each past interim value as the NPV of its subsequent cash flows (recall that this first scenario assumes no pre-determined, TWR-based estimates are available).

In each case, the discount rate should be the cost of capital rate. See equation (6) for the algebraic mechanics of computing NPVs.

With regard to scenario 2, justification of the use of the IRR is problematic for the following reason. In order to reach some comfort-level that the IRR result is even close to the correct rate of return for a project, one would need to compute its specific sequence of implied interim project values and review them for reasonableness. But if one is already thinking of 'checking' the IRR's implied sequence of interim project values for reasonableness, then this implies that there are some estimates as to what the interim market values really are. A truly valid rate of return metric should use those estimates.

Fortunately, such a rate of return metric now exists. It was introduced in Magni (2010) and it can use those aforementioned market values, or best estimates of them, directly. The AIRR allows users to specify what the sequence of the project's interim values is and computes a rate of return using those specific values. Specifically, AIRR is just that: the investment's return for each unit of capital invested. It can be equivalently viewed as the weighted average of the project's periodic rates of return, each of which is the period's income as a percentage of its interim values. The weights are based on the NPVs of the interim project values, each discounted at the cost of capital.

As a simple illustration, consider an annual cash flow sequence of $-\$100$, $\$0$, $+\$144$. The IRR of this cash flow is undoubtedly 20%, but is that 20% figure also really the "rate of return" of the actual underlying investment? If the project was worth $\$120$ after the first year, as implied by the IRR function, then the answer is yes. Otherwise, the project's rate of return is something else. For example, if the interim value is $\$110$, the rate of return in the first year is $10\% = (110 - 100)/100$, which is the return on $\$100$ of invested capital, and the rate of return in the second year is $30.91\% = (144 - 110)/110$, which is the return on $\$110$ of invested capital. Therefore, the true rate of return is summarized by an average of the two rates, weighted by the present values of the amount invested. The resulting rate, which depends on the

cost of capital, is in general different from 20% (e.g., closer to 21% with an 8% cost of capital).

The definition of AIRR is as follows: For a T -period project with cash flow stream $(CF_0, CF_1, \dots, CF_T)$, denoting with V_t the true (i.e., marketplace) interim project value as of time t and with

$$I_t = CF_t + V_t - V_{t-1} \quad (1)$$

the corresponding income, the project's period rate of return is:

$$i_t = \frac{I_t}{V_{t-1}} \quad (2)$$

and the AIRR is computed as the weighted average:

$$AIRR = \frac{i_1 C_0 + i_2 PV(V_1) + \dots + i_T PV(V_{T-1})}{C_0 + PV(V_1) + \dots + PV(V_{T-1})}, \quad (3)$$

where $C_0 = -F_0$ and $PV(V_t)$ is the present value of the interim project value at time t and the sum of these $PV(V_t)$ terms is just the NPV of the (market-based) interim project values. By averaging across each period's rate of return, AIRR makes it obvious that 'interim' values matter, something that is all too easy to forget with the IRR which, as seen, implicitly devises its own interim values that it is a rate of return *on*. It is worthwhile to note that equation (3) is simply a return on capital, which is the natural definition of a rate of return.

The mechanics of the AIRR computation are presented in Appendix 2, where more detailed examples are presented. In one example, the correct rate of return is 22.7% versus an IRR of 20.0%. In another example, the correct rate of return is lower, rather than higher, than the IRR result. It can go either way.

While the above definition makes the AIRR intuitive (it is a weighted average of the project's periodic rates of return), there is a computational, completely equivalent, shortcut that enables the user to apply a simplified formula (thereby avoiding the computation of incomes and rates of return detailed in the exhibits in Appendix 2). Better still,

this shortcut formula works even in those unlikely instances where some of the interim project values are zero in some period(s):

$$AIRR = COC + \frac{NPV \text{ of cash flows} \times (1 + COC)}{NPV \text{ of interim investment values}} \quad (4)$$

[see Magni (2010, p. 160) for this formula].⁵

It is worth noting that the second term on the right-hand side of (4) is an excess rate of return and, with a simple manipulation, we get:

$$\begin{aligned} & NPV \text{ of interim cash flows} \\ &= NPV \text{ of interim investment values} \\ &\times \frac{AIRR - COC}{1 + COC}. \end{aligned} \quad (4a)$$

In such a way, one realizes that the NPV of the investment's cash flows is an overall excess return, where the capital invested is the NPV of interim investment values and $(AIRR - COC)$ is the 'above-normal' rate of return.

Summary of AIRR's Data Requirements

Owing to the equation (4) just presented, the computation of AIRR is simple. AIRR requires users to commit to: Requirement 1—specifying a cost of capital, and Requirement 2—supplying interim investment values.

Requirement 1: Specifying a cost of capital. For ex ante purposes, projecting going-forward rates of return of various investments without appropriate (risk-adjusted) cost of capital thresholds is folly. However one might hope to avoid it, when a pro-forma results in a rate of return that seems borderline, one will need to decide where to draw the line; and that line is the cost of capital. And even in instances when the rate of return seems very high or very low, one must have an implicit estimate of what the cost of capital is, otherwise one could not say that the IRR is 'high' or 'low' (a rate of return is high or low only with respect to some implicit or explicit 'normal' rate of return). Also, to assess projects with different risks, one has to necessarily use different costs of capital reflecting the risk of the project.

For ex post purposes, the argument for having a cost of capital may seem less clear. Indeed, some performance reporting analysts may maintain that a rate of return is just a statistic, devoid of any risk given that the rate of return is being measured after the fact. However, this too is folly. How would one compare one investment that achieved a high IRR but was subject to a high degree of risk to another investment that achieved a somewhat lower IRR but was subject to a much lower degree of risk? The answer is that one needs to have cost of capital values, properly risk-adjusted, regardless of whether the application is ex ante or ex post.

Requirement 2: Supplying interim investment values. For ex ante applications, as alluded to earlier, for those who are unwilling to engage in explicit estimation, they can rely on a simple ‘automatic’ procedure, which provides more economically meaningful interim values than the IRR’s implied values. The procedure involves computing the NPV of future projected cash flows, discounted at the market-based cost of capital.

This procedure is described algebraically as follows: The IRR-implied interim values, denoted by C_t (assuming a constant rate of return) are given by:

$$C_t = \sum_{k=t+1}^n \frac{CF_k}{(1+i)^{k-t}} \quad t = 1, 2, \dots, n-1 \quad (5)$$

where i is the IRR (see Appendix 1). However, as discussed earlier, the IRR rate is an invalid rate for estimating interim market values, so the appropriate correction is to replace the IRR with the cost of capital so as to obtain the *theoretically* correct market value V_t :

$$V_t = \sum_{k=t+1}^n \frac{CF_k}{(1+COC)^{k-t}} \quad t = 1, 2, \dots, n-1. \quad (6)$$

This provides all of the interim value estimates necessary to compute the NPV result in the denominator of equation (4).

For ex post applications, i.e., for performance reporting purposes, if one has already produced periodic interim values for purposes of computing

TWRs, then those are the values that should be used for AIRR. The existence of such pre-determined interim values makes the case for AIRR all the more compelling.⁶ In cases where one did not produce such values, seemingly because the investor did not require regular progress updates at all, one can utilize the exact same procedure as was just recommended above in the ex ante case (i.e., one should compute the NPV of the subsequent cash flows). In effect, one would treat the subsequent cash flows as “future” cash flows. For example, the appropriate estimated value in year 8 for an asset that has produced a cash flow of \$11 in year 9 and \$121 upon sale in year 10, would be $\$11/(1.1) + \$121/(1.1^2) = \$10 + \$100 = \$110$, i.e., if the cost of capital were 10%.⁷

So, in sum, with cost of capital and interim values in hand, the only thing the evaluator needs to do is apply the simple formula in (4) above and the result will be a new and improved MWR metric that has none of the problems of the IRR. Specifically, there is always an easy-to-compute, closed-form AIRR, and it is always unique, as well as consistent with NPV.

Conclusion

In general, the IRR is merely the interest rate solution to a NPV equation that is set equal to zero. If the IRR is to be something more than that (i.e., if it is to be a true “rate of return”), then it must necessarily be a rate of return on something (i.e., ‘certain values’). Very few users likely understand what those certain values are. Most simply believe that, one way or another, the IRR magically produces a “rate of return.” As it turns out, these certain values are chosen by the IRR function itself, hence, the “internal” descriptor in the IRR moniker. But these values are unreliable, because they are arbitrarily determined and disconnected from market values. Indeed, the IRR is a rate of return based on implied values that are the result of past cash flows that should not, in any rational sense, affect current value.

Logic seems to dictate that, if the user has better estimates of the interim values of his or her holdings, then the rate of return metric chosen should

reflect those values, rather than using an IRR metric that defaults to its own internal choices for these interim values. Fortunately, with the creation of the AIRR metric in 2010, this is now possible.

When computing rates of return from an *ex ante* perspective, where estimations of the correct interim values may be sometimes difficult, even if the analyst is unwilling to engage in an explicit estimation procedure, one may fruitfully compute the estimated values in an automated fashion, by discounting subsequent cash flows at the cost of capital. In this way, the values obtained are the values implicitly determined by the cost of capital. However approximate they may be, they will, in the large majority of cases, turn out to be more accurate than the ones implicitly determined by the IRR.

When computing rates of return *ex post* (i.e., for performance reporting purposes), to the extent that users are already producing interim values for the purpose of calculating TWRs of return or for reporting values to clients, needing to use these better values for AIRR is no burden whatsoever. Indeed, it could be inferred that the use of the IRR is equivalent to producing a second set of values that disturbingly contradict the best estimates that are being used for TWRs, assuming that TWRs are being calculated.⁸ If no such periodic estimates have been made, the user can easily estimate interim values by discounting subsequent cash flows at the cost of capital.

Fortunately, the AIRR metric has none of the problems of the IRR: it always exists, is unique, as well as consistent with NPV analysis [see Magni (2010, 2011) for details]. The AIRR itself is easy to program. In fact, if we did not have powerful computers with “canned” functions, it would be far easier to compute AIRR than it would be to iteratively solve for the IRR.⁹

Rather than using the IRR, which employs an arbitrary estimation of interim values, the investor now has the choice of either explicitly estimating the interim values or using the cost of capital’s implied interim values, in order to obtain a more reliable rate of return. That rate of return is the AIRR.

Appendix 1

Boulding (1935) seems to have been the first person to formally create the notion of the IRR. His discussion starts with the notion of capital: capital is periodically invested in a project and generates a rate of return equal to i . The author assumes that the rate is constant per-period and also notes that it reflects an “internal” valuation. He then states that, during each period, the capital grows at this fixed (internal) rate and, at the end of the period, that amount is diminished (or augmented) by the cash flow that is distributed to (or contributed by) the investor, likening it to a bank account whose balance changes each period as additional monies are deposited or withdrawn. Algebraically, each such current value of capital C_t is related to the prior value C_{t-1} and the current end of period cash flow CF_t , as follows:

$$C_1 = C_0(1 + i) - CF_1 \quad (\text{A.1})$$

$$C_2 = C_1(1 + i) - CF_2 \quad (\text{A.2})$$

and so on, until the terminal capital value:

$$C_T = C_{T-1}(1 + i) - CF_T \quad (\text{A.3})$$

is obtained. However, the terminal capital is zero, or, to put it equivalently, the value of the project, after it has been liquidated, is zero. So, $C_T = 0$. Using (A.1)–(A.3) and noticing that the initial capital invested is $C_0 = -CF_0$, one obtains:

$$C_T = CF_0(1 + i)^T + CF_1(1 + i)^{T-1} + CF_2(1 + i)^{T-2} + \cdots + CF_T = 0,$$

which becomes:

$$\begin{aligned} \frac{C_T}{(1 + i)^T} &= CF_0 + \frac{CF_1}{1 + i} + \frac{CF_2}{(1 + i)^2} \\ &+ \cdots + \frac{CF_T}{(1 + i)^T} = 0. \end{aligned} \quad (\text{A.4})$$

Operationally, once (A.4) is solved for i , the interim capitals are automatically determined by (A.1)–(A.3). Hence, the IRR equation does not merely provide a rate of return, but also a companion implied sequence of interim capital (C_0, C_1, \dots),

C_{T-1}). It is also worthwhile noting that, owing to the condition $C_T = 0$, (A.3) becomes:

$$C_{T-1} = \frac{CF_T}{1+i}$$

$$C_{T-2} = \frac{C_{T-1} + CF_{T-1}}{(1+i)} = \frac{CF_{T-1}}{(1+i)} + \frac{CF_T}{(1+i)^2}$$

$$C_{T-3} = \frac{C_{T-2} + CF_{T-2}}{(1+i)} = \frac{CF_{T-2}}{(1+i)} + \frac{CF_{T-1}}{(1+i)^2} + \frac{CF_T}{(1+i)^3}$$

and, in general,

$$C_t = \frac{C_{t+1} + CF_{t+1}}{(1+i)} = \frac{CF_{t+1}}{(1+i)} + \dots + \frac{CF_T}{(1+i)^{T-t}},$$

which is just equation (5). Evidently, C_t has nothing to do with market values, for it depends on the IRR. The IRR in turn depends on all cash flows, even the past ones. So, C_t depends on the past cash flows, whereas market value only depends on expectations.

Appendix 2

Scenario 1

For purposes of illustration, consider a hypothetical development project where an ongoing influx of capital is required up until the date when it is completed, at which point it is subsequently sold. For simplicity, assume annual cash flows such as are indicated in column A of Exhibit 2A. The IRR of these cash flows is 20%. Column B denotes the implied sequence of interim project values that the 20% is a rate of return 'on,' according to the IRR as indicated by the approach used by Boulding (1935).¹⁰ In essence, this 'constant force' IRR approach assumes enough appreciation is earned so that the same 20% rate of return is generated each year, after accounting for the impact of the additional contribution.

Suppose, however, that reality is such that the project's valuation increases more evenly over time. For sake of simplicity, as more capital is invested, suppose that the actual property values appreciate from the initial \$100 to the final \$1254.9¹¹

at a constant growth rate¹² as shown in Exhibit 2A, column C. Clearly, these values are much lower than the ones in column B, suggesting that the rate of return on such a lower base, will surely be higher. And, indeed, using the AIRR method, assuming a cost of capital of 8%, we find that the correct rate of return is about 22.7%, rather than 20.0%. The algorithm necessary to produce the AIRR can be ascertained by referring to the Column Indicator row in Exhibit 2A.

It is worth noting that, using the shortcut described in equation (4), the computations in columns D to H in Exhibit 2A are not even needed, and the answer is given simply as follows:

$$AIRR = 8\% + \frac{NPV \text{ of cash flows} \times (1 + 8\%)}{NPV \text{ of investment values}}$$

$$= 8\% + \frac{290.67 \times (1 + 8\%)}{2138.4} = 22.68\%.$$

Scenario 2

Next, consider a hypothetical operating asset with fluctuating cash flows from one year to the next, perhaps due to expiring tenancies that leave space temporarily vacant before getting filled. Suppose the cash flows are as indicated in the column A of Exhibit 2B. It can easily be shown that the IRR of these cash flows is 20%. Column B denotes the implied sequence of interim project values that the 20% is a rate of return 'on,' according to the IRR as indicated by the constant force IRR approach used by Boulding (1935). As one can readily observe from the periodic changes in column B, whenever there is a large distribution in column A, the IRR implicitly assumes that the value of the remaining asset either grows very little or even decreases. Alternatively, whenever there is a small, or no, distribution, the IRR assumes that the value of the remaining asset grows substantially. In essence, the IRR assumes that the same rate of return is earned each period with some of it coming in the form of a distribution, and the rest, if any, coming in the form of an increase in the remaining value of the underlying asset.

Suppose, however, that reality is such that the variation in annual distributions is due to operating

cash flow changes that merely reflect temporary vacancies in the property (or capital improvement expenditures); and that these short-term variations do not markedly affect the overall value of the property. For example, the value of the property might increase rather evenly over time, similar to what one might expect if it were valued with a net present value methodology over, say, the next ten year holding period. Basically, we are assuming that the property has some low rate of return years and some high rate of return years, as a result of large real-world variations in operating cash flow. For simplicity, suppose the actual property values appreciate from the initial \$100 to the final \$176.4,¹³ again with a constant rate of growth pattern, as illustrated in column C of Exhibit 2B. Clearly, these values are much different than the ones in column B and are virtually always higher, suggesting that the rate of return on such a higher base, will surely be less. And, indeed, using the AIRR method, assuming a cost of capital of 8%,

we find that the correct rate of return is about 19.25%, rather than 20%. The algorithm necessary to produce the AIRR can be ascertained by referring to the “Column Indicator” row in Exhibit 2 below.

Using the shortcut, one immediately gets the same answer:

$$\begin{aligned} \text{AIRR} &= 8\% + \frac{\text{NPV of cash flows} \times (1 + 8\%)}{\text{NPV of investment values}} \\ &= 8\% + \frac{81.59 \times (1 + 8\%)}{783.3} = 19.25\% \end{aligned}$$

Admittedly, this second example is a bit extreme and no doubt concocted, but there is little doubt that operating cash flow can vary significantly from year to year, especially if the cash flow being analyzed is net of debt, and for a variety of reasons that do not have a marked effect on market value.

Exhibit 2A
Comparison of IRR and AIRR Values and Breakdown of AIRR Methodology

| IRR Approach | | | AIRR Approach—Constant Appreciation Rate = 37.2% | | | | | |
|------------------|-------------------|-------------------------------|--|---------------------|---------------------|-----------------------|------------------------------|---|
| Year | Cashflows | Interim Values Implied by IRR | Actual Market-based Values ^a | Period Appreciation | Period Total Income | Period Rate of Return | PV of Beginning Market Value | Product of Two Prior Columns |
| | –\$100.0 | \$100.0 | \$100.0 | | | | | |
| 1 | –\$50.0 | \$170.0 | \$137.2 | \$37.2 | –\$12.8 | –12.8% | \$100.0 | –\$12.8 |
| 2 | –\$50.0 | \$254.0 | \$188.2 | \$51.0 | \$1.0 | 0.7% | \$127.0 | \$0.9 |
| 3 | –\$50.0 | \$354.8 | \$258.2 | \$70.0 | \$20.0 | 10.6% | \$161.4 | \$17.1 |
| 4 | –\$50.0 | \$475.8 | \$354.3 | \$96.0 | \$46.0 | 17.8% | \$205.0 | \$36.5 |
| 5 | –\$50.0 | \$620.9 | \$486.0 | \$131.8 | \$81.8 | 23.1% | \$260.4 | \$60.1 |
| 6 | –\$50.0 | \$795.1 | \$666.8 | \$180.8 | \$130.8 | 26.9% | \$330.8 | \$89.0 |
| 7 | –\$50.0 | \$1,004.1 | \$914.7 | \$248.0 | \$198.0 | 29.7% | \$420.2 | \$124.8 |
| 8 | \$1,204.9 | \$0.0 | \$0.0 | –\$914.7 | \$290.2 | 31.7% | \$533.7 | \$169.3 |
| Total | NPV@8% = \$290.67 | NPV@8% = \$2,616 | NPV@8% = \$2,138.4 | | \$754.9 | | \$2,138.4 | \$485.0 |
| Column Indicator | A | B | C | D = C – Prior C | E = A + D | F = E / Prior C | G = PV (Prior C) | H = F * G |
| | IRR = 20.00% | | | | | | | AIRR = Col H Total / Col G Total = 22.68% |

Note: This table uses cashflows based on a hypothetical development project.

^aAssuming constant appreciation rate.

Exhibit 2B

Comparison of IRR and AIRR Values and AIRR Methodology

| IRR Approach | | | AIRR Approach—Constant Appreciation Rate = 7.3% | | | | | |
|------------------|------------------|-------------------------------|---|---------------------|---|-----------------------|------------------------------|------------------------------|
| Year | Cashflows | Interim Values Implied by IRR | Actual Market-based Values ^a | Period Appreciation | Period Total Income | Period Rate of Return | PV of Beginning Market Value | Product of Two Prior Columns |
| | –\$100.0 | \$100.0 | \$100.0 | | | | | |
| 1 | \$25.0 | \$95.0 | \$107.3 | \$7.3 | \$32.3 | 32.3% | \$100.0 | \$32.3 |
| 2 | \$0.0 | \$114.0 | \$115.2 | \$7.9 | \$7.9 | 7.3% | \$99.4 | \$7.3 |
| 3 | \$28.0 | \$108.8 | \$123.7 | \$8.5 | \$36.5 | 31.6% | \$98.8 | \$31.3 |
| 4 | \$0.0 | \$130.6 | \$132.8 | \$9.1 | \$9.1 | 7.3% | \$98.2 | \$7.2 |
| 5 | \$31.0 | \$125.7 | \$142.6 | \$9.8 | \$40.8 | 30.7% | \$97.6 | \$30.0 |
| 6 | \$0.0 | \$150.8 | \$153.0 | \$10.5 | \$10.5 | 7.3% | \$97.0 | \$7.1 |
| 7 | \$34.0 | \$147.0 | \$164.3 | \$11.2 | \$45.2 | 29.6% | \$96.4 | \$28.5 |
| 8 | \$176.4 | \$0.0 | \$0.0 | –\$164.3 | \$12.1 | 7.3% | \$95.9 | \$7.0 |
| Total | NPV@8% = \$81.59 | NPV@8% = \$734.4 | NPV@8% = \$783.3 | | \$194.4 | | \$783.3 | \$150.8 |
| Column Indicator | A | B | C | D = C – Prior C | E = A + D | F = E/Prior C | G = PV (Prior C) | H = F * G |
| IRR = 20.00% | | | | | AIRR = Col H Total/Col G Total = 19.25% | | | |

Note: This table uses cashflows based on a hypothetical operating asset.

^aAssuming constant appreciation rate.

Endnotes

1. Actually, the metric they deal with is but a version of the well-known Modified Internal Rate of Return (Lin, 1976; Shull, 1993).
2. Multiple IRR solutions can wreak havoc with incentive fee schemes although, if one uses a “preferred return formulation” as discussed in Altshuler and Schneiderman (2011), the incentive fee problems can be circumvented.
3. Since IRR is merely the interest rate solution to a discounted cash flow equation, it is easy to forget that, in most cases, it is also being interpreted as a rate of return. Indeed, it seems that most users of IRR are unaware of what IRR is a rate of return on, some thinking it is a rate of return on the initial contribution, others thinking it is a rate of return on all contributions, still others thinking it is a rate of return on all contributions net of distributions. None of these is quite right.
4. Fisher’s (1930) earlier ‘rate of return over cost’ can also be considered an internal rate of return, associated with an incremental investment alternative [see Alchian (1955) for the conceptual relationship between the two metrics].
5. The distracting $(1 + COC)$ factor in (4) and (4a) is necessary only to align the two NPVs, so that the numerator’s ‘income’ is one period ahead of the denominator’s ‘investment,’ as one expects when computing a rate of return. Alternatively, this factor can be eliminated as long as the point is made that the numerator NPV is then measured as of the beginning of period one, whereas the denominator NPV is measured as of the beginning of period zero.
6. If the desired cash flow periodicity is more frequent than the valuation frequency, then the IRR can still play a role. Specifically, if a TWR procedure produced quarterly valuations, but the desired cash flow periodicity were to be daily, then AIRR would require daily valuations. The best interpolated values on those days between each quarter’s beginning and end would be those implied by an IRR for that single quarter, i.e., assuming a constructive purchase at the beginning of the quarter and a constructive sale at the end of the quarter.
7. Such a procedure, using a pre-specified cost of capital, would eliminate any gaming of the AIRR in those occasional instances where a manager is asked to produce a track record retrospectively, using periodic values that were never estimated as the quarters passed.
8. Although it is often noted that dollar-weighted and time-weighted returns are different, in truth, their differences are meant to merely reflect the assumed lack of control, in the case of TWR, of the manager over interim contributions and distributions. The intention is that they should be identical in every other respect, especially with regard to interim project valuations. Differing interim project values represent an unintended consequence of using IRR as the dollar-weighted metric.
9. IRR and XIRR are solved only by trial and error, and not always easily, as evidenced by the fact that, sometimes, initial guesses need to be ‘spot on’ for software systems to find a solution, assuming one exists; and hopefully, it is the only one, something that spreadsheet software systems, for example, do not tell the user.

10. As seen in the Appendix 1, these values are exactly the NPV of the future cash flows, discounted at the 20% IRR rate.
11. The final value is set equal to the final cash flow plus an add-back of an additional \$50 M contribution assumed to be made in that period, just prior to sale.
12. A constant rate of appreciation in the interim values is assumed simply to concoct a scenario that a reader can easily recreate, aware that the exhibit's values are rounded off. The use of such an assumption does not, in any way, intend to suggest that interim values actually grow at a constant rate; indeed, they never really do.
13. The final value is set equal to the final cash flow under the alternating year assumption that the final year's operating cash flow is due to be zero again.

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